

**NAME**

**qmath** - fixed-point math library based on the "Q" number format

**SYNOPSIS**

```
#include <sys/qmath.h>
```

**DESCRIPTION**

The **qmath** data types and APIs support fixed-point math based on the "Q" number format. The APIs have been built around the following data types: *s8q\_t*, *u8q\_t*, *s16q\_t*, *u16q\_t*, *s32q\_t*, *u32q\_t*, *s64q\_t*, and *u64q\_t*, which are referred to generically in the earlier API definitions as *QTYPE*. The *ITYPE* refers to the `stdint(7)` integer types. *NTYPE* is used to refer to any numeric type and is therefore a superset of *QTYPE* and *ITYPE*.

This scheme can represent Q numbers with [2, 4, 6, 8, 16, 32, 48] bits of precision after the binary radix point, depending on the *rpsht* argument to **Q\_INI()**. The number of bits available for the integral component is not explicitly specified, and implicitly consumes the remaining available bits of the chosen Q data type.

Operations on Q numbers maintain the precision of their arguments. The fractional component is truncated to fit into the destination, with no rounding. None of the operations is affected by the floating-point environment.

For more details, see the *IMPLEMENTATION DETAILS* below.

**LIST OF FUNCTIONS****Functions which create/initialise a Q number**

<i>Name</i>	Description
<b>Q_INI(3)</b>	initialise a Q number

**Numeric functions which operate on two Q numbers**

<i>Name</i>	Description
<b>Q_QADDQ(3)</b>	addition
<b>Q_QDIVQ(3)</b>	division
<b>Q_QMULQ(3)</b>	multiplication
<b>Q_QSUBQ(3)</b>	subtraction
<b>Q_NORMPREC(3)</b>	normalisation
<b>Q_QMAXQ(3)</b>	maximum function
<b>Q_QMINQ(3)</b>	minimum function
<b>Q_QCLONEQ(3)</b>	

identical copy

Q\_QCPYVALQ(3)

representational copy

### **Numeric functions which apply integers to a Q number**

<i>Name</i>	Description
Q_QADDI(3)	addition
Q_QDIVI(3)	division
Q_QMULI(3)	multiplication
Q_QSUBI(3)	subtraction
Q_QFRACI(3)	fraction
Q_QCPYVALI(3)	overwrite

### **Numeric functions which operate on a single Q number**

<i>Name</i>	Description
Q_QABS(3)	absolute value
Q_Q2D(3)	double representation
Q_Q2F(3)	float representation

### **Comparison and logic functions**

<i>Name</i>	Description
Q_SIGNED(3)	determine sign
Q_LTZ(3)	less than zero
Q_PRECEQ(3)	compare bits
Q_QLTQ(3)	less than
Q_QLEQ(3)	less or equal
Q_QGTQ(3)	greater than
Q_QGEQ(3)	greater or equal
Q_QEQ(3)	equal
Q_QNEQ(3)	not equal
Q_OFLOW(3)	would overflow
Q_RELPREC(3)	relative precision

### **Functions which manipulate the control/sign data bits**

<i>Name</i>	Description
Q_SIGNSHFT(3)	sign bit position
Q_SSIGN(3)	sign bit

**Q\_CRAWMASK(3)**  
                   control bitmask  
**Q\_SRAWMASK(3)**  
                   sign bitmask  
**Q\_GCRAW(3)** raw control bits  
**Q\_GCVAL(3)** value of control bits  
**Q\_SCVAL(3)** set control bits

### Functions which manipulate the combined integer/fractional data bits

<i>Name</i>	Description
<b>Q_IFRAWMASK(3)</b>	integer/fractional bitmask
<b>Q_IFVALIMASK(3)</b>	value of integer bits
<b>Q_IFVALFMASK(3)</b>	value of fractional bits
<b>Q_GIFRAW(3)</b>	raw integer/fractional bits
<b>Q_GIFABSVAL(3)</b>	absolute value of fractional bits
<b>Q_GIFVAL(3)</b>	real value of fractional bits
<b>Q_SIFVAL(3)</b>	set integer/fractional bits
<b>Q_SIFVALS(3)</b>	set separate integer/fractional values

### Functions which manipulate the integer data bits

<i>Name</i>	Description
<b>Q_IRAWMASK(3)</b>	integer bitmask
<b>Q_GIRAW(3)</b>	raw integer bits
<b>Q_GIABSVAL(3)</b>	absolute value of integer bits
<b>Q_GIVAL(3)</b>	real value of integer bits
<b>Q_SIVAL(3)</b>	set integer bits

### Functions which manipulate the fractional data bits

<i>Name</i>	Description
<b>Q_FRAWMASK(3)</b>	fractional bitmask
<b>Q_GFRAW(3)</b>	raw fractional bits
<b>Q_GFABSVAL(3)</b>	

absolute value of fractional bits  
 Q\_GFVAL(3) real value of fractional bits  
 Q\_SFVAL(3) set fractional bits

### Miscellaneous functions/variables

<i>Name</i>	Description
Q_NCBITS(3)	number of reserved control bits
Q_BT(3)	C data type
Q_TC(3)	casted data type
Q_NTBITS(3)	number of total bits
Q_NFCBITS(3)	number of control-encoded fractional bits
Q_MAXNFBITS(3)	number of maximum fractional bits
Q_NFBITS(3)	number of effective fractional bits
Q_NIBITS(3)	number of integer bits
Q_RPSHFT(3)	bit position of radix point
Q_ABS(3)	absolute value
Q_MAXSTRLEN(3)	number of characters to render string
Q_TOSTR(3)	render string
Q_SHL(3)	left-shifted value
Q_SHR(3)	right-shifted value
Q_DEBUG(3)	render debugging information
Q_Dfv2BFV(3)	convert decimal fractional value

### IMPLEMENTATION DETAILS

The **qmath** data types and APIs support fixed-point math based on the "Q" number format. This implementation uses the Q notation  $Qm.n$ , where  $m$  specifies the number of bits for integral data (excluding the sign bit for signed types), and  $n$  specifies the number of bits for fractional data.

The APIs have been built around the following `q_t` derived data types:

```
typedef int8_t      s8q_t;
typedef uint8_t     u8q_t;
typedef int16_t     s16q_t;
typedef uint16_t    u16q_t;
typedef int32_t     s32q_t;
typedef uint32_t    u32q_t;
```

```
typedef int64_t      s64q_t;
typedef uint64_t     u64q_t;
```

These types are referred to generically in the earlier API definitions as *QTYPE*, while *ITYPE* refers to the stdint(7) integer types the Q data types are derived from. *NTYPE* is used to refer to any numeric type and is therefore a superset of *QTYPE* and *ITYPE*.

The 3 least significant bits (LSBs) of all q\_t data types are reserved for embedded control data:

- bits 1-2 specify the binary radix point shift index operand, with 00,01,10,11 == 1,2,3,4.
- bit 3 specifies the radix point shift index operand multiplier as 2 (0) or 16 (1).

This scheme can therefore represent Q numbers with [2,4,6,8,16,32,48,64] bits of precision after the binary radix point. The number of bits available for the integral component is not explicitly specified, and implicitly consumes the remaining available bits of the chosen Q data type.

Additionally, the most significant bit (MSB) of signed Q types stores the sign bit, with bit value 0 representing a positive number and bit value 1 representing a negative number. Negative numbers are stored as absolute values with the sign bit set, rather than the more typical two's complement representation. This avoids having to bit shift negative numbers, which can result in undefined behaviour from some compilers.

This binary representation used for Q numbers therefore comprises a set of distinct data bit types and associated bit counts. Data bit types/labels, listed in LSB to MSB order, are: control ‘C’, fractional ‘F’, integer ‘I’ and sign ‘S’. The following example illustrates the binary representation of a Q20.8 number represented using a s32q\_t variable:

M	L
S	S
B	B
3322222222221111111111	
10987654321098765432109876543210	
SIIIIIIIIIIIIIIIIIIIIIIFFFFFFFFFFCCC	

Important bit counts are: total, control, control-encoded fractional, maximum fractional, effective fractional and integer bits.

The count of total bits is derived from the size of the `q_t` data type. For example, a `s32q_t` has 32 total bits.

The count of control-encoded fractional bits is derived from calculating the number of fractional bits per the control bit encoding scheme. For example, the control bits binary value of 101 encodes a fractional bit count of  $2 \times 16 = 32$  fractional bits.

The count of maximum fractional bits is derived from the difference between the counts of total bits and control/sign bits. For example, a `s32q_t` has a maximum of  $32 - 3 - 1 = 28$  fractional bits.

The count of effective fractional bits is derived from the minimum of the control-encoded fractional bits and the maximum fractional bits. For example, a `s32q_t` with 32 control-encoded fractional bits is effectively limited to 28 fractional bits.

The count of integer bits is derived from the difference between the counts of total bits and all other non-integer data bits (the sum of control, fractional and sign bits.) For example, a `s32q_t` with 8 effective fractional bits has  $32 - 3 - 8 - 1 = 20$  integer bits. The count of integer bits can be zero if all available numeric data bits have been reserved for fractional data, e.g., when the number of control-encoded fractional bits is greater than or equal to the underlying Q data type's maximum fractional bits.

## EXAMPLES

### Calculating area of a circle with `r=4.2` and `rpshft=16`

```
u64q_t a, pi, r;
char buf[32]

Q_INI(&a, 0, 0, 16);
Q_INI(&pi, 3, 14159, 16);
Q_INI(&r, 4, 2, 16);

Q_QCLONEQ(&a, r);
Q_QMULQ(&a, r);
Q_QMULQ(&a, pi);

Q_TOSTR(a, -1, 10, buf, sizeof(buf));
printf("%s\n", buf);
```

### Debugging

Declare a `Q20.8 s32q_t` number `s32`, initialise it with the fixed-point value for  $5/3$ , and render a debugging representation of the variable (including its full precision decimal C-string representation), to the console:

```

s32q_t s32;
Q_INI(&s32, 0, 0, 8);
Q_QFRACI(&s32, 5, 3);
char buf[Q_MAXSTRLEN(s32, 10)];
Q_TOSTR(s32, -1, 10, buf, sizeof(buf));
printf(Q_DEBUG(s32, "", "\n\ttostr=%s\n", 0), buf);

```

The above code outputs the following to the console:

```

"s32"@0x7ffffffe7d4
      type=s32q_t, Qm.n=Q20.8, rpsht=11, imin=0xfff00001, \
imax=0xffff
      qraw=0x00000d53
      imask=0x7fff800, fmask=0x000007f8, cmask=0x00000007, \
ifmask=0x7fffff8
      iraw=0x00000800, iabsval=0x1, ival=0x1
      fraw=0x00000550, fabsval=0xaa, fval=0xaa
      tostr=1.664

```

Note: The "\" present in the rendered output above indicates a manual line break inserted to keep the man page within 80 columns and is not part of the actual output.

## SEE ALSO

errno(2), math(3), Q\_FRAWMASK(3), Q\_IFRAWMASK(3), Q\_INI(3), Q\_IRAWMASK(3), Q\_QABS(3), Q\_QADDI(3), Q\_QADDQ(3), Q\_SIGNED(3), Q\_SIGNSHFT(3), stdint(7)

## HISTORY

The **qmath** functions first appeared in FreeBSD 13.0.

## AUTHORS

The **qmath** functions and this manual page were written by Lawrence Stewart <lstewart@FreeBSD.org> and sponsored by Netflix, Inc.